Mathe 1 (Algebra) - Formeln

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October 4, 2004

1 Lineare Abbildung α

$\alpha(0) = 0$
$\alpha(-x) = -\alpha(x)$
$\alpha(x + y) = \alpha(x) + \alpha(y)$
$\alpha(\lambda y) = \lambda\alpha(y)$
$\alpha^{-1} \circ \alpha = \alpha \circ \alpha^{-1} = id$

Assoziativgesetz	$\alpha + (\beta + \gamma) = (\alpha + \beta) + \gamma$	$(\alpha \circ \beta) \circ \gamma = \alpha \circ (\beta \circ \gamma)$
Distributivgesetz	$\alpha \circ (\beta + \gamma) = \alpha \circ \beta + \alpha \circ \gamma$	$(\beta + \gamma) \circ \alpha = \beta \circ \alpha + \gamma \circ \alpha$

2 Lineare Abbildung $\Leftrightarrow$ Matrix

$\alpha(x)$	Ax
$(\alpha + \beta)(x)$	$(A + B)x$
$(\lambda\alpha)(x)$	$(\lambda A)x$
$(\beta \circ \alpha)(x)$	$(\beta(\alpha(x))) = (BA)x$
α *bijektiv*	A invertierbar
$\rightarrow \alpha^{-1}(x)$	$\rightarrow A^{-1}x$

3 Matrizen

Drehmatrix $D = \begin{pmatrix} cos\varphi & -sin\varphi \\ sin\varphi & cos\varphi \end{pmatrix}$	Spiegelmatrix $S = \begin{pmatrix} cos\varphi & sin\varphi \\ sin\varphi & -cos\varphi \end{pmatrix}$

Assoziativgesetz	$A + (B + C) = (A + B) + C$	$(AB)C = A(BC)$
Distributivgesetz	$A(B + C) = AB + AC$	$(B + C)A = BA + CA$

$A^{-1}A = AA^{-1} = 1_n$
$(A^{-1})^{-1} = A$
$(AB)^T = B^T A^T$
$(AB)^{-1} = B^{-1}A^{-1}$
$(A^T)^{-1} = (A^{-1})^T$

4 Permutationen

Transposition τ	Permutation α
$\tau \circ \tau = id$	$\alpha = \tau_1 \circ ... \circ \tau_m$
$\tau^{-1} = \tau$	$\alpha^{-1} = \tau_m \circ ... \circ \tau_1$
$sgn(\tau) = -1$	$sgn(\alpha) = (-1)^m$
	$sgn(\alpha \circ \beta) = (sgn\alpha)(sgn\beta)$

5 Determinante einer n×n-Matrix

$det(A) = |A| := \prod_{i=1}^{n} a_{ii}$ für Dreiecksmatrix A
$det(AB) = (detA)(detB)$
$detA \neq 0 \Leftrightarrow A \in GL(n, K) \Leftrightarrow$ A invertierbar
$detA^T = detA$
$detA^{-1} = \frac{1}{detA}$
$det(\lambda A) = \lambda^n detA$
$det(A) = -det(A_{Z_1 - Umformung})$
$det(A_{Nullzeile}) = det(A_{Nullspalte}) = 0$
$det \begin{pmatrix} A & * \\ 0 & B \end{pmatrix} = detA \cdot detB$
$AdjA = (\det(A) \cdot A^{-1})^T$

6 Polynome

Polynom $A(X) = \sum_{i}^{n} a_i X^i$; $a_i \in K$	$gradA = n$
$\lambda A(X) = \sum_{i} (\lambda a_i) X^i$	$grad\lambda A = gradA$
$(A + B)(X) = \sum_{i} (a_i + b_i) X^i$	$grad(A + B) \leq max\{gradA, gradB\}$
$(AB)(X) = \sum_{i} c_i X^i$ mit $c_i = \sum_{k=0}^{i} a_k b_{i-k}$	$gradAB = gradA + gradB$, wenn $A \neq 0 \neq B$

7 Komplexe Zahlen

Koordinatenform	Polarform
$z = a + ib$, a=Re z, b=Im z $\in \mathbf{R}$	$z = \|z\|(cos\varphi + isin\varphi)$, $0 \leq \varphi = arg(z) < 2\pi$
$i^2 = -1$, $i^{-1} = -i$, $\|z\| = \sqrt{a^2 + b^2} \geq 0$	
$z_1 z_2 = (a_1 a_2 - b_1 b_2) + i(a_1 b_2 + b_1 a_2)$	$z_1 z_2 = (\|z_1\|\|z_2\|, \varphi_1 + \varphi_2)$
$z_1 + z_2 = (a_1 + a_2) + i(b_1 + b_2)$	$z^n = (\|z\|^n, n\varphi)$
	$\sqrt[n]{z} = (\sqrt[n]{\|z\|}, \frac{\varphi + 2k\pi}{n})$; $k = 0, .., n-1$
$\overline{z} = a - ib$	$\overline{z} = (\|z\|, 2\pi - \varphi)$

$\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$
$\overline{z_1 z_2} = \overline{z_1} \cdot \overline{z_2}$
$z\overline{z} = |z|^2$
$|z_1 z_2| = |z_1| \cdot |z_2|$
$|z_1 + z_2| \leq |z_1| + |z_2|$
$z^{-1} = \frac{\overline{z}}{|z|^2}$, für $z \neq 0$
$|\frac{z}{|z|}| = 1$, für $z \neq 0$

8 Abstand und Orthogonalität

Euklidisches Skalarprodukt

in $\mathbf{R}$	in $\mathbf{C}$
$[x, y] = \sum_{i=1}^{n} x_i y_i$	$[x, y] = \sum_{i=1}^{n} x_i \overline{y_i}$
$[x, y] = [y, x]$	$[x, y] = \overline{[y, x]}$
$[\lambda x + \mu z, y] = \lambda[x, y] + \mu[z, y]$	analog
$[x, \lambda y + \mu z] = \lambda[x, y] + \mu[x, z]$	$[x, \lambda y + \mu z] = \overline{\lambda}[x, y] + \overline{\mu}[x, z]$

$[x, x] \geq 0$; $[x, x] = 0 \Leftrightarrow x = 0$

$\|x\| = \sqrt{[x, x]}$	Betrag
$\|x - y\| = \sqrt{[x - y, x - y]}$	Abstand
$\|[x, y]\| \leq \|x\| \cdot \|y\|$	Cauchy-Schwarzsche Ungleichung
$\|x + y\| \leq \|x\| + \|y\|$	Dreiecksungleichung

$cos\measuredangle(x, y) = \frac{[x,y]}{\|x\|\|y\|}$
$x \perp y \Leftrightarrow [x, y] = 0$
Pythagoras $\forall i \neq j : v_i \perp v_j \rightarrow |v_1 + ... + v_r|^2 = |v_1|^2 + ... + |v_r|^2$