Mathe 2 (Analysis) - Formeln

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1 konvergente Folgen, Grenzwerte

Sei $\lim_{n\to\infty} a_n = a$; $\lim_{n\to\infty} b_n = b$:
$\lim_{n\to\infty} f(a_n) = f(a)$, falls f stetig
$\lim_{n\to\infty} a_n \pm b_n = a \pm b$
$\lim_{n\to\infty} a_n \cdot b_n = a \cdot b$
$\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{a}{b}; b \neq 0, \forall n : b_n \neq 0$
$\lim_{n\to\infty} (\lambda a_n) = \lambda a$
für fast $\forall n : a_n \leq b_n \to a \leq b$
l'Hospital : $\lim_{x\to x_0} \frac{f(x)}{g(x)} = \lim_{x\to x_0} \frac{f'(x)}{g'(x)} = c; x_0 \in \mathbf{R} \cup \infty$, falls c existiert

2 bestimmte Folgen und Grenzwerte

$\lim_{n\to\infty} \frac{\sin n}{n} = \lim_{n\to\infty} \frac{\cos n}{n} = 0$ (Ü2/A3f)
$\lim_{x\to 0} \frac{\sin x}{x} = 1$ (Ü4/A8)
$\lim_{x\to 0} \frac{\cos x}{x} = \infty$
$\lim_{n\to\infty} \sqrt[n]{n!} = \infty$ (Ü2/A3g)
$\lim_{n\to\infty} \sqrt[n]{n} = 1$
$\lim_{n\to\infty} \sqrt[n]{\alpha} = 1$, $\alpha \in \mathbf{R}^+$
$\lim_{n\to\infty} (1 + \frac{\alpha}{n})^n = e^\alpha$, $\alpha \in \mathbf{R}$

3 Reihen

$$\sum_{k=0}^{\infty}(a_k \pm b_k) = \left(\sum_{k=0}^{\infty} a_k\right) \pm \left(\sum_{k=0}^{\infty} b_k\right)$$

$$\sum_{k=0}^{\infty}(\lambda a_k) = \lambda \sum_{k=0}^{\infty} a_k$$

$$\exp(x) = \sum_{k=0}^{\infty} \frac{x^k}{k!}, x \in \mathbf{R}$$

$$\cos(x) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!}, x \in \mathbf{R}$$

$$\sin(x) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}, x \in \mathbf{R}$$

Geometrische Reihe $\sum_{k=0}^{\infty} q^k = \begin{cases} \frac{1}{1-q} & |q| < 1, q \in \mathbf{R} \\ \infty & q \geq 1 \\ divergent & sonst \end{cases}$

Harmonische Reihe $\sum_{k=1}^{\infty} \frac{1}{k} = \infty$

4 Sonstiges: e, ln, sin(h), cos(h)

$$e^{x+y} = e^x \cdot e^y$$

$$e^{ix} = \cos(x) + i\sin(x)$$

$$e^{z+2\pi i} = e^z$$

$\sin x = \frac{1}{2i}(e^{ix} - e^{-ix})$	$\sinh x = \frac{1}{2}(e^x - e^{-x})$
$\cos x = \frac{1}{2}(e^{ix} + e^{-ix})$	$\cosh x = \frac{1}{2}(e^x + e^{-x})$
$\cos^2(x) + \sin^2(x) = 1$	$\cosh^2(x) - \sinh^2(x) = 1$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\sin(x+y) = \sin(x)\cos(y) - \cos(x)\sin(y)$$

$$\sin(x) - \sin(y) = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos(x) - \cos(y) = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$e^{\ln x} = x$$

$$\ln(xy) = \ln(x) + \ln(y)$$

$$\ln\left(\frac{x}{y}\right) = \ln(x) - \ln(y)$$

$$\ln(x^n) = n\ln(x)$$

$$\log_a(x) = \frac{log_b(x)}{log_b(a)}$$

5 Ableitungen

$(x^n)' = n \cdot x^{n-1}; \forall n \in \mathbf{R}$

$(f+g)'(x_0) = f'(x_0) + g'(x_0)$ (Summenregel)

$(\lambda f)'(x_0) = \lambda f'(x_0)$ (Homogenität)

$(fg)'(x_0) = f'(x_0)g(x_0) + f(x_0)g'(x_0)$ (Produktregel)

$\left(\frac{f}{g}\right)'(x_0) = \frac{f'(x_0)g(x_0) - f(x_0)g'(x_0)}{(g(x_0))^2}$ (Quotientenregel)

$(g(f(x_0)))' = g'(f(x_0)) \cdot f'(x_0)$ (Kettenregel)

$(f^{-1}(y_0))' = \frac{1}{f'(x_0)} = \frac{1}{f'(f^{-1}(y_0))}$ (Umkehrfunktion)

6 Integration

$\int (f(x) + g(x))dx = \int f(x)dx + \int g(x)dx$

$\int \lambda f(x)dx = \lambda \int f(x)dx$

Partielle Integration: $\int f(x)g'(x)dx = f \cdot g - \int f'(x)g(x)dx$

Substitution: $\int f(\varphi(x)) \cdot \varphi'(x)dx = \left(\int f(u)du\right)_{u=\varphi(x)}$

7 bestimmte Integrale

$\int_a^b f(x)dx \leq \int_a^b g(x)dx; \forall x \in [a,b] : f(x) \leq g(x)$

$\int_a^b f(x)dx = -\int_b^a f(x)dx$

$\int_a^a f(x)dx = 0$

$\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$

$\int_a^b f(\varphi(x))\varphi'(x)dx = \int_{\varphi(a)}^{\varphi(b)} f(u)du$ (Integration durch Substitution)

8 uneigentliche Integrale

$\int_0^1 \frac{1}{x^\alpha}dx = \begin{cases} \frac{1}{1-\alpha} & \alpha < 1 \\ \infty & \alpha \geq 1 \end{cases}$	$\int_0^1 x^\alpha dx = \begin{cases} \frac{1}{\alpha+1} & \alpha > -1 \\ \infty & \alpha \leq -1 \end{cases}$
$\int_1^\infty \frac{1}{x^\alpha}dx = \begin{cases} \frac{1}{\alpha-1} & \alpha > 1 \\ \infty & \alpha \leq 1 \end{cases}$	$\int_1^\infty x^\alpha dx = \begin{cases} -\frac{1}{\alpha+1} & \alpha < -1 \\ \infty & \alpha \geq -1 \end{cases}$