Mathe 4 (WR1) - Wahrscheinlichkeitsverteilungen

Thomas Schneider

March 4, 2006

S: Skript, A: Übungsaufgaben, M: Materialsammlung

1 diskrete Wahrscheinlichkeitsverteilungen

Laplace Verteilung $\mathcal{L}(\Omega)$ S37	$f(\omega) = \frac{1}{\lvert\Omega\rvert}$	
Hypergeometrische Verteilung $\mathcal{H}(N, K, n)$ S37, M51	$f(k) = \frac{\binom{K}{k} \cdot \binom{N-K}{n-k}}{\binom{N}{n}}$ $1 \leq K \leq N; 1 \leq n \leq N$	$m_1 = \frac{K}{N} n$
Binomial Verteilung $\mathcal{B}(n, p)$ S38	$f(k) = \binom{n}{k} p^k q^{n-k}$ $0 < p < 1; q = 1 - p; p, q \in \mathbf{R}$	
Geometrische Verteilung $\mathcal{G}(p)$ S38	$f(n) = pq^{n-1}$ $0 < p < 1; q = 1 - p$	$m_1 = \frac{1}{p}$ $\hat{m_2} = \frac{q}{p^2}$ $M(t) = \frac{pe^t}{1-qe^t}$
Poisson-Verteilung $\mathcal{P}(\mu)$ S40	$f(n) = e^{-\mu} \frac{\mu^n}{n!}$ $\mu \in \mathbf{R}^+$	$m_1 = \mu$ $\hat{m_2} = \mu$ $M(t) = e^{\mu(e^t - 1)}$
Pascal-Verteilung A8	$f(k) = \frac{a^k}{(1+a)^{k+1}}$ $a > 0; k \in \mathbf{N_0}$	$m_1 = a$ $\hat{m_2} = a(1 + a)$ $M(t) = \frac{1}{(1 + a)(1 - e^t \frac{a}{1+a})}$

2 absolutstetige Verteilungen

uniforme Verteilung $\mathcal{U}(M)$ S66	$f(x) = \frac{1}{\lambda(M)} \cdot 1_M(x) = \begin{cases} \frac{1}{\lambda(M)} & , x \in M \\ 0 & , \text{sonst} \end{cases}$	
uniforme Verteilung auf Intervall $\mathcal{U}[a,b]$ S74	$f(t) = \begin{cases} 0 & , t < a \\ \frac{1}{b-a} & , a \le t \le b \\ 0 & , t > b \end{cases}$ $F(t) = \begin{cases} 0 & , t < a \\ \frac{t-a}{b-a} & , a \le t \le b \\ 1 & , t > b \end{cases}$	
standardisierte Normalverteilung $\mathcal{N}(0,1)$ S67	$\varphi(t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2}$ $\phi(t) = \int_{-\infty}^{t} \varphi(x)dx$	$m_1 = 0$ $\hat{m}_2 = 1$
Normalverteilung $\mathcal{N}(\mu,\sigma^2)$ S126	$f(t) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(t-\mu)^2}{2\sigma^2}}$ $F(t) = \int_{-\infty}^{t} f(x)dx$	$m_1 = \mu$ $\hat{m}_2 = \sigma^2$
Exponentialverteilung $\mathcal{E}(\lambda)$ S74	$f(t) = \begin{cases} \lambda e^{-\lambda t} & , t > 0 \\ 0 & , t \le 0 \end{cases}$ $F(t) = \begin{cases} 1 - e^{-\lambda t} & , t > 0 \\ 0 & , t \le 0 \end{cases}$ $\lambda > 0$	$m_1 = \frac{1}{\lambda}$ $\hat{m}_2 = \frac{1}{\lambda^2}$ $M(t) = \frac{\lambda}{\lambda - t}$
Rayleigh-Verteilung S85	$f(t) = \begin{cases} \frac{t}{\beta^2} e^{-\frac{1}{2}(\frac{t}{\beta})^2} & , t > 0 \\ 0 & , t \le 0 \end{cases}$ $F(t) = \begin{cases} 1 - e^{-\frac{1}{2}(\frac{t}{\beta})^2} & , t > 0 \\ 0 & , t \le 0 \end{cases}$ $\beta > 0$	$m_1 = \beta\sqrt{\frac{\pi}{2}}$ $\hat{m}_2 = \beta^2(2 - \frac{\pi}{2})$
Cauchy-Verteilung A 15	$f(x) = \frac{1}{\pi} \frac{\alpha}{\alpha^2 + (x-\beta)^2}$ $\alpha > 0; \beta \in \mathbf{R}$	
Extremwertverteilung A16	$f_a(t) = e^{-(t-a)} e^{-e^{-(t-a)}}$ $F_a(t) = e^{-e^{-(t-a)}}$ $a \in \mathbf{R}$	$m_1 = a + e^{-(t-a)}$
Erlang-Verteilung Grad 2 S100	$f(t) = \begin{cases} \lambda^2 t e^{-\lambda t} & , t > 0 \\ 0 & , t \le 0 \end{cases}$	
"Beim (n+1)-ten Wurf 2. Treffer" M36	$f(n) = np^2q^{n-1}$ $n = 1, 2, ...; 0 < p < 1; q = 1 - p$	$m_1 = 1 + 2\frac{q}{p}$ $M(t) = p^2 \frac{e^t}{(1 - e^t q)^2}$
Sonstige 1 M61	$f(x) = \begin{cases} \frac{\alpha\beta}{\alpha - \beta}(e^{-\beta x} - e^{-\alpha x}) & , x > 0 \\ 0 & , x \le 0 \end{cases}$ $0 < \alpha < \beta$	
Sonstige 2 M62	$f(x) = \begin{cases} 6x(1-x) & , 0 \le x \le 1 \\ 0 & , \text{sonst} \end{cases}$	$m_1 = \frac{1}{2}$ $\hat{m}_2 = \frac{1}{20}$